Factor Allocation and Asset Allocation[1]

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Abstract:
We examine four different asset pricing factors and their use in a portfolio that varies over time based on an investor's risk preferences. Using data for the period 1980 to 2014, we show that the risk premiums of different factors are not constant over time, and that investors may improve their risk return trade-off by weighting or "tilting" their portfolios differently as liquidity and risk tolerances change such as when investors age. Our results suggest that those investors targeting higher returns should tilt towards the size and value factors, while investors favoring lower levels of risk should tilt towards the quality factor. Our results raise questions about the current industry approach to asset allocation and the driving forces behind the magnitude of risk premiums over time.

Key Words: Factor Pricing Model, Asset Pricing, Asset Allocation, Empirical Finance

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I. Introduction

Factor investing, sometimes called smart beta investing or fundamental indexing, has seen increased interest in the financial field in recent years. The consensus in contemporary investment research is that risk factors offer investors an opportunity to earn higher returns if they are willing to take on the risk associated with the underlying factors.[4] Many contemporary researchers examine which factors offer risk premiums and the sources of those risk premiums. Recent additions to this literature include the low volatility risk factor (Ang et al, 2006; Ang et al, 2009) and the profitability risk factor (Novy-Marx, 2014) for instance.

Much of the investment literature today is dominated by research testing the existence of various new asset pricing factors (e.g., the profitability factor of Novy-Marx) that explain existing market anomalies (e.g., the low vol anomaly of Ang et. al) or propose new factor models. In contrast, relatively little attention is paid to examining the relative variation in risk premiums paid to factors and how these risk premiums vary over time.

In addition, and perhaps more importantly from a practitioner's standpoint, there is little consensus on how investors should use those risk factors as part of a portfolio. While some research has been conducted examining the possibility of "timing" factors by adjusting exposure based on expectations of the future (e.g., Benos, Jochec, and Nyekel, 2010; Tarun and Shivakumar, 2002), there has been little work done on how an optimal portfolio might adjust factor exposure based on characteristics of the investor that change over time – most notably the investor's age. Aside from age of course, investors also have risk and liquidity needs that vary based on their personal life circumstances. Given the variation in investor needs, it is surprising

[4] Consistent with frequent industry practice, we define risk via standard deviation throughout this paper. While tracking error is sometimes used as a measure of risk in many investment strategies, different benchmarks are used for tracking error, which makes it more difficult to use as a risk proxy in this study.

that the extant literature has mostly failed to explain how factor investments should differ based on investor characteristics. Black and Litterman (1992) offer a model based on this concept, but to date it has been little applied to the factor investing arena.

This concept leads to a variety of unanswered questions: do risk premiums in factor models vary over time, and if so, what forces drive the level of compensation investors receive for taking on exposure to these risk premiums?

Current research has established firmly that pricing factors under the Carhart (1997) four-factor pricing model are widespread and present across asset classes. For instance, Asness, Moskowitz and Pedersen (2013) find evidence that value and momentum styles are prevalent across not only geographical areas in equities, but also asset classes including government bonds, currencies, and commodities. They also highlight the negative correlation between value and momentum and support the notion that the styles are therefore complimentary. Fisher, Shah, and Titman (2016) offer compelling evidence that a simple 50-50 mix of value and momentum outperforms a strategy using either style in isolation. Yet it remains unclear if the characteristics of these factor risk premiums are constant over time, or if investors should consider weighting towards either value or momentum based on their own personal investment objectives.

This question as it relates to the stationarity of risk premiums across time is fundamentally important for two reasons. First, it helps to improve financial economists' understanding of what risk premiums investors are willing to pay when taking on particular aspects of broader market risks.

Second, and perhaps more importantly, it establishes a novel approach to market timing in the literature. Market timing has traditionally been interpreted as being related to the alpha generated by particular portfolio managers or trading strategies. Yet some investors care less about four-factor alphas than about absolute returns and the risks they must take to achieve them. To the extent that a manager could tilt towards various factor premiums and help clients exposed to those factors generate a more attractive risk/return profile, then the manager would have an opportunity to provide valuable benefits to clients.

To the extent that financial advisors can play a role in helping clients to tilt towards factors that help align with the investor’s risk/return needs, the financial advisor can add crucial value. Many studies have shown that investors undermine their own investment performance through excessive trading, herding behavior, and attempts to time the market. Hence advisors who can help a client set an optimal portfolio and then stick with that portfolio can offer value to the investor.

There is some evidence on this front already, but it is largely specific to individual factors and often limited in scope. For instance, both Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2015) show that momentum strategies can be improved by considering the time-varying nature of volatility. Their intuition rests with higher volatility being predictive of future momentum crashes.

We examine evidence around returns and variation in returns for 14 different portfolios built around different weightings or “tilts” for four widely used factors by practitioners: size, value, quality, and momentum. Our results show marked differences in returns and risk between different portfolio formulations. These differences are economically meaningful and suggest that investors should not be using a one- size- fits- all approach to building a factor portfolio. Specifically, our results suggest that those investors targeting higher returns should tilt towards the size and value factors, while investors favoring lower levels of risk (i.e., a lower portfolio standard deviation) should tilt towards quality. A single investor might adopt different portfolio tilts over his lifetime as his risk preferences change– a finding that represents a challenge to the conventional view of portfolio management which rarely moves beyond a single diversified factor-based portfolio for all investors.

The remainder of this paper is organized as follows. In Section 2 we review the past literature in the area and develop our hypotheses; in Section 3 we describe our data and methodology; in Section 4 we present the empirical results; in Section 5 we conclude.

II. Background Literature

The asset pricing literature has a tumultuous and sometimes contentious history of competing models going back to the original Capital Asset Pricing Model (CAPM) and before. Factor-based models such as the three-factor and four-factor models have increasingly dominated the research landscape since the mid-1990s, especially as tests have shown that factor models are effective in explaining both time-series and cross-sectional stock returns (Ferson and Harvey, 1999). Factor models do not explain all market anomalies well, but variations on intertemporal CAPM have been unsuccessful as well (Lewellen, and Nagel, 2006). Thus, most research in this arena so far has focused on either intertemporal changes in CAPM loadings or on static estimates of multi-factor models. Little work has been done in examining a hybrid of these two strands of literature in particular with respect to whether multi-factor premiums vary dynamically over time.

Multi-factor models came about as a natural extension of the single-factor CAPM of Sharpe (1964). Arguably the most important empirical multi-factor model is that of Fama-French (1993).

$$R_i - R_{rf} = \alpha + \beta_m MRP + \beta_s SMB + \beta_v HML + \varepsilon \tag{1}$$

In equation (1) above[5], the return on a stock in excess of the risk-free return (typically monthly T-bills) is modeled as a linear function of the returns on three market-neutral zero-investment factors. The three factors are the market factor (MRP), a size factor (SmB), and the value factor (HmL). Since Fama and French (1993), the three -factor model has been augmented by additional factors. For example, Carhart (1996) added a momentum factor (UmD) in his study of mutual fund performance. Recently, Fama and French (2015) offer a five-factor model which adds Investment and Profitability factors to the original three factors. In general, we can describe an N-factor model to describe excess returns. Other factors shown to have explanatory power for excess returns are Profitability (Novy-Marx), Liquidity (Pastor and Stambaugh), and Quality (Asness et al.). There are also alternative factors seeking to capture the same underlying phenomenon. For example, there is the original Fama-French version of HmL, which use lagged market stock prices in its construction, while the Asness et. al version of HmL uses current stock prices.

One classic concern in this literature is that if the CAPM is inherently misspecified, then using time-varying estimates for beta may result in pricing errors (Ghysels, 1998). Subsequent research has used conditional skewness (Harvey and Siddique, 2000) and other techniques to refine conditional beta models while avoiding mispricing errors (Adrian and Franzoni, 2009). The asymmetric response of market volatility to shocks is well documented in the literature. The "leverage effect" explanation dates to Black (1975). More recent papers on the topic in the US markets include Engle and Ng (1993), Bekaert and Wu (2000), Wu (2001), and Dennis, Mayhew, and Stivers (2006), all showing that asymmetric volatility plays a role in pricing of risk

[5] The time subscripts are omitted to simplify notation.

premiums. The testing of asymmetric volatility uses GARCH modeling with an asymmetry term. Prominent examples of volatility modeling of stock returns include Nelson (1991) and Glosten, Jagannatahn, and Runkle (1993).

III. Hypotheses and Methodology

Our approach to the factor investment literature is to examine how an investor might tilt his portfolio based on stages of his life cycle over time. The extant academic literature is generally built around using a multifactor model that constructs portfolios based on breakpoints for a particular factor – e.g., investing in stocks simultaneously in the top tercile for momentum and smallest quintile for size. This academic approach implicitly equally weights all factors used in the investment process. Within industry, a more common practice is to assign each stock a score based on multiple factors, often based on the percentile rank a particular stock achieves for a particular factor, and then form a portfolio of stocks with the highest composite score across factors. The approach would lead an investor to buy stocks that score particularly well on any single factor and to also invest in stocks that simply score moderately well on all factors. Again, this process requires weighting factors – for instance, with an equal weighting, a high score on "value" is no more or less important than a high score on "momentum".

Since our interest is in examining more- effective methods of portfolio construction for industry, we follow the intuitive industry approach. Our process scores stocks based on four well- known factors – size, value, momentum, and quality using data from CRSP and Compustat. Our portfolio construction approach mirrors that of well-known portfolio management software packages used in industry. Once these scores are computed, we create composite scores by weighting each factor and then summing all four scores. Tables 1 and 2 in the Appendix illustrate that our individual factors are highly correlated with 12- month look- ahead stock returns, as is our composite score. Our approach relies on rolling windows for forming portfolios.

Next we compute a static equal-weighted four -factor portfolio by following a simple set of steps:

1.) Compute factor percentiles for size, value, momentum, and quality[6]

2.) Weight each factor - 25% each for static and variable weights for tilted portfolios

3.) Compute weighted average percentile across all four factors

4.) Form long-only EW portfolio of all stocks that score in the top 25%

In order to study the potential benefits from alternatives to equal weighting of factors, we construct portfolios giving greater weight to each factor in turn. Portfolios can be tilted towards one factor or two factors.

Our single- factor tilts are as follows:

One Factor Tilt 1: 40% to single factor, 20% to each of remaining 3 factors

One Factor Tilt 2: 70% to single factor, 10% to each of remaining 3 factors

We also perform two- factor tilts based on 40% to two factors, and 10% to each of the remaining two factors. This approach results in fifteen separate portfolios as shown below.

Static	25% weighting to all four factors
S1	40% to Size, 20% to remaining factors
S2	70% to Size, 10% to remaining factors
V1	40% to Value, 20% to remaining factors
V2	70% to Value, 10% to remaining factors
Q1	40% to Quality, 20% to remaining factors

[6] Quality is defined using the standard definition in the past factor literature where a quality score is based on stock metrics for safety, profitability, and earnings growth. Quality is highly correlated with the investment factor of Fama and French.

Q2	70% to Quality, 10% to remaining factors
M1	40% to Momentum, 20% to remaining factors
M2	70% to Momentum, 10% to remaining factors
SQ	80% to Size and Quality, 10% to remaining factors
SM	80% to Size and Momentum, 10% to remaining factors
SV	80% to Size and Value, 10% to remaining factors
VQ	80% to Value and Quality, 10% to remaining factors
VM	80% to Value and Momentum, 10% to remaining factors
QM	80% to Quality and Momentum, 10% to remaining factors

Each of the fifteen portfolios holds an independent set of stocks. The portfolios are rebalanced quarterly, based on data from the previous quarter.

IV. Results

Descriptive statistics are shown in Table 1 below. The results show that, while average (mean) returns between different portfolios are broadly similar, the standard deviations of returns differ more dramatically. Higher standard deviations make it less likely that an investor will be able to tolerate drawdowns and remain in the factor portfolio over time. The static portfolio (*static*), which uses a four- factor model that equally weights all four factors and does not change over time, has an average return of 15.62% annually for the sample period. The equal-weighted market portfolio returns an average of 13.06% during the same time frame.

By comparison, the lowest -performing tilted portfolio is the 70% Quality- tilted portfolio (Q2) with an average annual return of 12.86%. The best- performing portfolio is the 40% Size/40% value portfolio (SV), with an average return of 16.44%. While the 82 basis points in additional return for the SV portfolio versus the static portfolio is economically meaningful in the cross-section, on a time- series basis this difference is not statistically significant. Essentially, annual variation in returns between any single portfolio is far larger than the difference between any of the tilted portfolios in a given year.

The differences in standard deviations among portfolios are economically and statistically significant. The static and market portfolios have annual standard deviations of 44.6% and 37.0%, respectively. These results are broadly consistent with the notion that factor investing adds to expected returns in exchange for greater risk as encapsulated by standard deviation on the portfolio.

The tilted portfolios have annualized standard deviations (from monthly returns) that range on the low end from a below -market standard deviation of 11.6% (70% quality weighted portfolio – Q2) to 16.8% (70% size weighted portfolio – S2). The results for kurtosis are similar, with the S2 portfolio performing best on that metric.

Broadly speaking, one general takeaway from the descriptive statistics is that investors can optimize their portfolio in many ways depending on their preferences. Investors looking to maximize return would choose a factor portfolio with a different tilt than investors looking to minimize risk as measured by standard deviation or kurtosis (a proxy for crash risk).

Table 1: Descriptive Statistics (Mean, Variance, Skewness, Kurtosis): Maximum values of Mean Returns, and minimum values of standard deviation, skewness, and kurtosis are underlined

Portfolio	**Mean Return**	**Std. Dev**	**Skew**	**Kurtosis**
Static	*0.1562*	*0.1486*	*3.7864*	*26.8476*
S1	0.1617	0.15722	3.7076	25.0265
S2	0.1623	0.16703	3.6463	<u>23.4057</u>
V1	0.1566	0.1415	3.8363	28.2746
V2	0.162	0.13842	3.8926	29.3638
Q1	0.1561	0.14062	3.9176	29.1502
Q2	0.1286	<u>0.1158</u>	4.2051	36.7181
M1	0.1413	0.14973	3.7551	26.5365
M2	0.1288	0.15722	3.6566	24.9391
SQ	0.1615	0.15228	3.7597	25.9867
SM	0.1425	0.16388	3.6277	23.7565

SV	<u>0.1644</u>	0.1506	3.7206	25.9069
VQ	0.1624	0.1326	4.065	32.038
VM	0.1506	0.14593	3.7604	27.0509
QM	0.1391	0.14142	3.9163	29.2386
Tilted Avg.	**0.1513**	**0.14673**	**3.8191**	**27.6709**
Market	*0.1306*	*0.12337*	*3.8852*	*32.0917*

This preliminary evidence is suggestive of an opportunity for investors to benefit by altering the tilt of their factor exposure based on changing needs during their lifetimes. One possible tilting strategy, for example, might entail investors taking greater exposure to the size and value factors early in life to capitalize on their higher expected returns, while slowly tilting towards quality if the investor becomes more risk averse as they age or as other life circumstances change.

One concern about the preliminary results might be that they are a by-product of data snooping. Investors might be tempted to herd into those factors which have done the best in any given year in an effort to maximize returns. More broadly, the existing industry products focused on factor investing and "Smart Beta" already suffer from concerns about investors withdrawing money whenever such factor-based strategies underperform the market. Table 2 addresses this issue.

The table below illustrates year-by-year performance for the S&P 500, the equal-weighted CRSP market portfolio, a static four -factor portfolio, a return- maximization portfolio (SV), and a risk-minimization portfolio (Q2). The results show that while the S&P and the market as a whole outperform the factor portfolios with some regularity, it is rare for the returns on the static four- factor portfolio to exceed the returns on the tilted max- return portfolio.

Of the 34 years in the sample period, the CRSP equal -weighted market return exceeds the static four- factor return in 14 years (roughly 41% of the time). The market return outperforms the maximum -return (SV) portfolio in 12 of the 34 years (35%), while in 11 of the 34 (32%) years the static- factor portfolio beat the maximum -return (SV) portfolio. Table 2 below illustrates the performance of tilted factor models versus the market as a whole and against a time-invariant or “static” four factor portfolio.

Despite the annual fluctuations in relative performance between different portfolios, in 59% of cases when factor-based portfolios outperform the market, they do so by a substantial margin. The static- factor portfolio outperforms the market by 4.20% in years when it beats the market, compared to factor underperformance of 65 basis points in years where the market outperforms. Factor investing outperforms the market roughly 60% of the time during our sample period, while the market outperforms a factor model roughly 40% of the time.

The story is similar for static -factor portfolios compared to the tilted portfolio. When a static four- factor portfolio outperforms the max- return tilted portfolio, the static portfolio return is only 40 basis points higher. In the 68% of cases where the tilted maximum -return portfolio beats the static portfolio, it does so by 2.85% annually for the 34 -year sample period.

Table 2: Year By Year Tilted Performance vs. S&P

Year	*S&P 500*	*Mkt*	*Static*	*Tilted (Max Ret - SV)*	*Tilted (Min Risk - Q2)*	*3-month T.Bill*	*10-year T. Bond*
1980	31.74%	23.55%	10.95%	20.09%	13.22%	11.22%	-2.99%
1981	-4.70%	-1.48%	5.15%	4.77%	-0.57%	14.30%	8.20%
1982	20.42%	63.68%	56.57%	62.83%	41.01%	11.01%	32.81%
1983	22.34%	-8.38%	1.68%	5.34%	0.89%	8.45%	3.20%
1984	6.15%	11.24%	7.34%	7.76%	12.36%	9.61%	13.73%
1985	31.24%	19.66%	11.89%	12.98%	15.74%	7.49%	25.71%
1986	18.49%	7.09%	10.03%	12.02%	10.15%	6.04%	24.28%
1987	5.81%	-7.84%	3.35%	1.15%	2.49%	5.72%	-4.96%
1988	16.54%	13.85%	13.12%	12.59%	16.42%	6.45%	8.22%
1989	31.48%	-8.95%	-5.41%	-5.70%	-0.35%	8.11%	17.69%
1990	-3.06%	21.27%	20.02%	18.18%	15.01%	7.55%	6.24%

1991	30.23%	25.81%	31.46%	32.40%	23.45%	5.61%	15.00%
1992	7.49%	23.17%	30.37%	32.87%	26.59%	3.41%	9.36%
1993	9.97%	6.49%	13.93%	15.54%	8.96%	2.98%	14.21%
1994	1.33%	14.24%	13.65%	14.49%	11.17%	3.99%	-8.04%
1995	37.20%	24.17%	19.73%	21.17%	19.88%	5.52%	23.48%
1996	22.68%	15.19%	17.72%	18.42%	20.41%	5.02%	1.43%
1997	33.10%	9.14%	11.84%	11.50%	12.66%	5.05%	9.94%
1998	28.34%	10.00%	14.15%	13.71%	2.73%	4.73%	14.92%
1999	20.89%	24.46%	14.93%	17.29%	8.27%	4.51%	-8.25%
2000	-9.03%	-1.16%	12.72%	11.97%	14.83%	5.76%	16.66%
2001	-11.85%	1.17%	11.20%	12.53%	11.97%	3.67%	5.57%
2002	-21.97%	24.37%	40.96%	38.69%	19.21%	1.66%	15.12%
2003	28.36%	47.38%	55.78%	60.10%	36.17%	1.03%	0.38%
2004	10.74%	11.40%	12.82%	13.35%	15.02%	1.23%	4.49%
2005	4.83%	15.60%	14.93%	15.34%	14.22%	3.01%	2.87%
2006	15.61%	7.90%	6.07%	6.98%	6.84%	4.68%	1.96%
2007	5.48%	-15.54%	-2.14%	-3.86%	-14.49%	4.64%	10.21%
2008	-36.55%	-24.73%	-21.45%	21.43%	-10.73%	1.59%	20.10%
2009	25.94%	41.74%	50.98%	48.24%	33.16%	0.14%	-11.12%
2010	14.82%	15.35%	7.44%	7.95%	9.42%	0.13%	8.46%
2011	2.10%	1.11%	3.61%	3.70%	3.61%	0.03%	16.04%
2012	15.89%	25.79%	22.41%	22.67%	20.76%	0.05%	2.97%
2013	32.15%	50.79%	47.87%	60.65%	33.65%	0.07%	-9.10%

One implication of these results is that unless investors can effectively forecast market returns versus factor returns, they are better off defining an investment strategy and sticking to it.[7] This analysis does not consider trading costs or tax implications, though those effects would tend to favor a defined investment strategy rather than a timing approach.

In order to attempt to maximize returns, some investors may be tempted to approach factor investing as an exercise akin to stock picking – by trying to make tactical allocations to different factors based on expectations about returns to those factors. Such an approach ignores the

[7] There is no convincing evidence that most investors can time the market and substantial evidence that efforts to do so result in poor performance (see for instance Bhattacharya, Loos, and Hackethal, 2016).

benefits of diversification among factors. Tables 3A and 3B below show that monthly returns among tilted portfolios and the market are relatively low. Hence, investors are choosing between investment portfolios that are not directly moving in sync with one another.

Table 3A: Single Focused Factor Tilt Return Correlations
Panel A:

	Static	S1	S2	V1	V2	Q1	Q2	M1
S1	0.820	1.000						
S2	0.581	0.755	1.000					
V1	0.834	0.728	0.518	1.000				
V2	0.668	0.578	0.413	0.795	1.000			
Q1	0.807	0.690	0.480	0.728	0.574	1.000		
Q2	0.330	0.269	0.156	0.305	0.228	0.520	1.000	
M1	0.796	0.690	0.484	0.686	0.523	0.679	0.269	1.000
M2	0.540	0.472	0.345	0.445	0.321	0.448	0.140	0.741

Table 3B: Two Factor Focused Tilt Return Correlations
Panel B:

	Static	SQ	SM	SV	VQ	VM	QM
SQ	0.676	1.000					
SM	0.709	0.562	1.000				
SV	0.729	0.654	0.583	1.000			
VQ	0.642	0.536	0.367	0.578	1.000		
VM	0.753	0.433	0.638	0.611	0.562	1.000	
QM	0.620	0.465	0.556	0.371	0.503	0.604	1.000

Thus far, the statistics presented suggest a possible benefit for investors from a tilted portfolio, but the economic magnitude of that benefit is unclear. To assess that benefit, we examine the long-term returns from a pair of tilting strategies.

In the first strategy, the investor starts off with 100% of allocation to the max -return tilted portfolio (SV weighted 40% size, 40% value, 10% quality, 10% momentum) and then the investor shifts 3% of assets each year into a minimum -risk portfolio (Q2 weighted 70% to quality, and 10% each to value, momentum, and size). Thus, at the end of the 34- year period, the investor is fully invested in the risk- minimizing portfolio. We label this strategy “Tilted Min SD Growth” in the figure below.

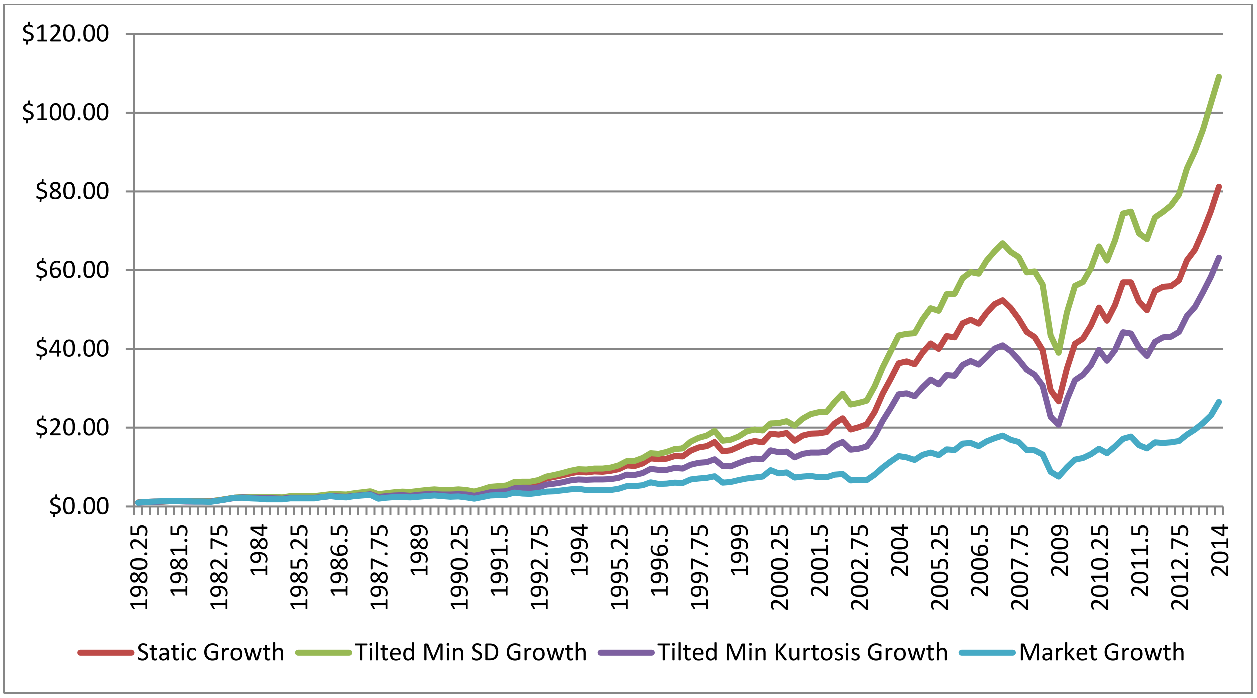
In the second strategy, which we label “Tilted Min Kurtosis Growth,” the investor again begins with 100% in the max -return tilted portfolio SV and then shifts 3% annually to the kurtosis -minimizing portfolio (S2 weighted 70% to size, and 10% each to value, momentum, and quality).

The results of these tilted life-cycle based approaches are significant economically by any measure. Investing $1 in a simple equal -weighted market portfolio starting in 1980 generates $26.52 by December 31, 2013. Tilted Min Kurtosis Growth portfolio turns $1 in 1980 into $63.17 by December 31, 2013. The static equal -weighted four- factor portfolio generates $81.19 in capital by December 31, 2013, while the Tilted Min SD Growth portfolio results in the highest final capital value of $109.90.[8]

The growth in the Tilted Min SD Growth portfolio represents an internal rate of return of 15.3% versus a 10.4% IRR for the simple equal -weighted market portfolio.

Figure 1: Growth of a Dollar

[8] Our results are robust to different intermediate starting points in the data.

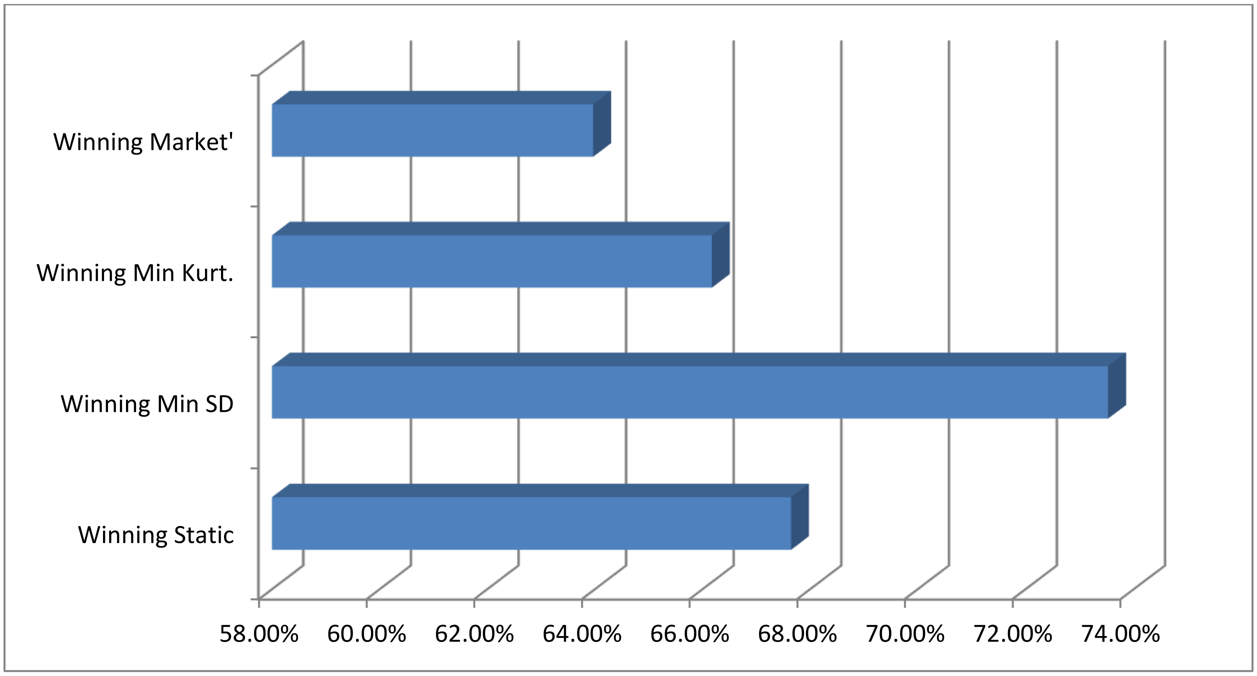


The tilted portfolios based on a potential life- cycle investment approach yield not only impressive cumulative growth of $1 versus the market, but the approach also offers an attractive option for investors interested in great consistency in portfolio growth, as Figure 2 below shows.

The figure below illustrates the frequency with which the portfolio produces a quarterly return greater than 0%. The market offers positive returns in just less than 64% of quarters during the 34- year period studied. The static four- factor portfolio achieves positive returns in 67.25% of quarters during that same time frame, while the Tilted Min SD Growth portfolio that starts by investing in the maximum -return portfolio and shifts towards a risk- minimizing portfolio over time offers positive returns in 73.52% of quarters. Hence, factor -tilting appears to offer investors the opportunity for both greater returns and fewer instances of negative returns during the period from 1980 to 2014.

Sharpe ratios for each of the four strategies offer a similar picture. The market's Sharpe ratio for the period is 0.29, compared to 0.46 for a static four -factor portfolio; 0.53 for the portfolio shifting from maximum returns to minimum standard deviation (Tilted Min SD Growth); and 0.42 for the portfolio shifting from maximum returns to minimum kurtosis (Tilted Min Kurtosis Growth).

Figure 2: Frequency of Returns Greater than 0%



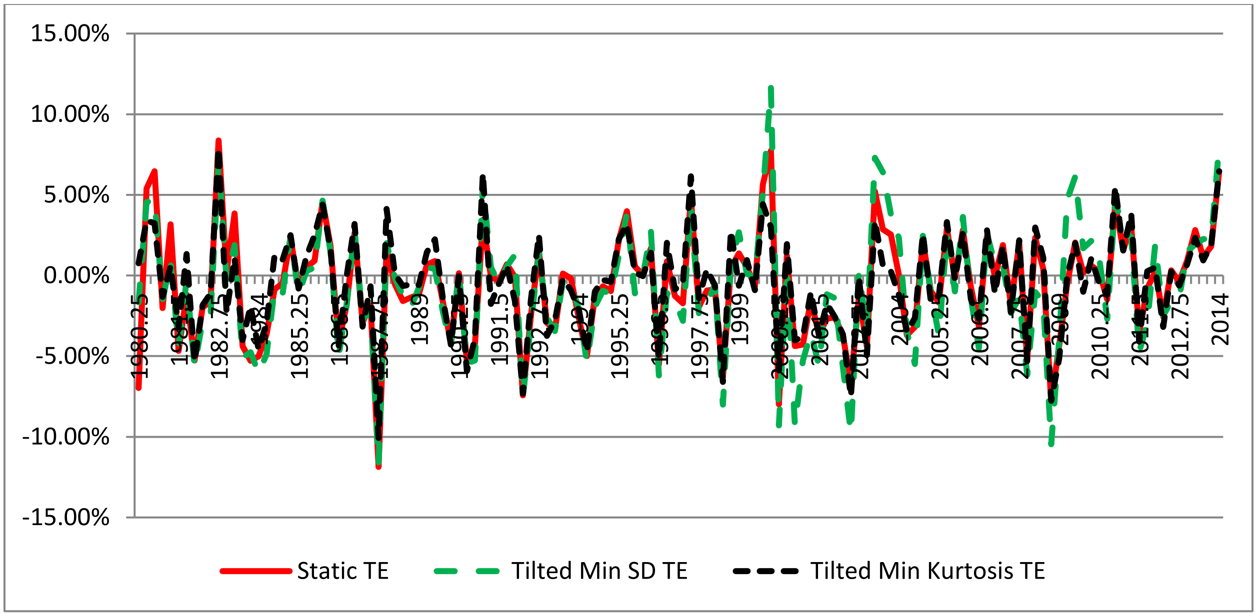
Finally, we examine tracking error for the static four -factor portfolio and the two tilting strategies versus the market. The figure below illustrates the difference in returns between the market and each of the factor-based strategies. The strategies outperform and underperform at similar times, as the figure shows graphically. The standard deviation of these differences is the tracking error on each strategy.

The tracking error for each strategy is as follows:

	Static Factor Tracking Error	Tilted Min SD Tracking Error	Tilted Min Kurtosis Tracking Error
Std. Dev	3.33%	3.86%	3.10%

While the tracking errors for all three strategies are relatively modest, the Tilted Min Kurtosis portfolio has a lower tracking error than either the static factor -portfolio or the Tilted Min SD Tracking Error portfolio. That result is largely because this portfolio minimizes exposure to periods of significant underperformance versus the market at the cost of returns over time.

Figure 3: Tracking Errors



	Static TE	Tilted Min SD TE	Tilted Min Kurtosis TE
Average	-0.62%	-0.80%	-0.47%

V. Conclusion

Overall, our results call into question a central assumed tenet of modern investing-- that there is a single optimal factor-based portfolio that investors should adopt. Instead, our results suggest that just as investors need to adopt different asset allocations, different factor allocations based on risk preferences may also benefit different investors as well. While the holdings in that portfolio may change over time, the assumed consensus has been that the factors should in essence be weighted equally, or at least consistently across investors. Our results show that, in fact, the returns and standard deviations for even modestly tilted portfolios vary markedly. A portfolio focused on maximizing returns will tilt towards size and value and offers dramatically higher returns than a simple equal- weighted portfolio. Moreover, a portfolio that starts out maximizing returns and then gradually shifts towards minimizing standard deviations of returns (by tilting towards quality) dramatically outperforms a static factor -based portfolio over the course of our sample period.

Overall, our results suggest that practitioners and academics alike should consider the characteristics of factor returns including mean, standard deviation, and kurtosis when designing portfolios to be used by different classes of investors.

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Appendix Table 1: Factors as derived are highly related to stock returns

Ret12	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Size_Decile	-.0136647	.000246	-55.55	0.000	-.0141469	-.0131825
Bk2Mkt_Decile	.0151638	.0002372	63.94	0.000	.0146989	.0156286
MOM_Decile	-.0015966	.0002206	-7.24	0.000	-.0020289	-.0011642
Qual_Avg_Decile	.0187158	.0003374	55.46	0.000	.0180544	.0193772
_cons	.0239785	.0027813	8.62	0.000	.0185273	.0294296

Appendix Table 2: Combined Factor Score is Highly Related to Stock Returns

Ret12	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Stock_Score	.0423797	.0004305	98.44	0.000	.0415359	.0432234
_cons	-.0587061	.0020224	-29.03	0.000	-.06267	-.0547422